J. Fluid Mech. (1980), *vol.* 100, *part* 2, *pp.* 225–236
Printed in Great Britain

Wave-tank experiments on an immersed vertical circular duct

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(Received 8 November 1979 and in revised form 22 February 1980)

A series of experiments is described in which a fully-submerged circular duct situated with its axis vertical is subjected to regular incident waves. The resulting wave-interaction effects are defined in terms of reflexion coefficients derived from wave-height measurements, and a pressure coefficient derived from measurements of pressure in the depths of the enclosure. The experiments were conducted in a wide tank, so simulating open-sea conditions, and in a narrow tank where wall effects were important. The particular case when a transverse standing wave was induced above the duct was examined in detail.

These three-dimensional experiments complement a previous investigation into the performance of two-dimensional ducts and are of current practical significance in the context of sub-sea wave-energy conversion.

1. Introduction

A series of experiments has been described recently in which a fully-submerged parallel-plate duct was subjected to regular incident waves (Knott & Flower 1979). That research was undertaken as part of a wider study into the performance of submerged, ocean-wave energy converters and followed the appearance of a theoretical study of the problem by Lighthill (1979).

In such an arrangement, a gravitationally-balanced fluid column, contained within a U-shaped duct, is forced to oscillate resonantly in response to wave-induced pressures occurring at its mouth. This mouth is open to the ambient water conditions while the other end of the tube vents to a constant-pressure air reservoir.

Lighthill considered in detail the two-dimensional case where the duct is considered to extend uniformly in the plane normal to incident waves. In this example the duct was represented by two parallel plates lying entirely beneath the surface and extending downwards to an infinite depth. A theoretical estimate was made of the wave reflexions and induced pressures, from which it was predicted that a substantial amplification of pressures within the enclosure occurred at certain wave frequencies; this phenomenon is favourable to the operation of the proposed devices.

The results of the aforementioned experiments bore out this theoretical prediction and provided overall support for the analytical approach at low wave amplitudes.

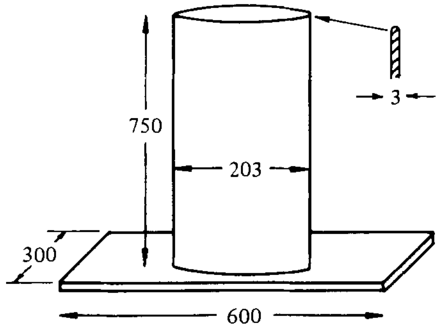

FIGURE 1. Circular aluminium duct, all measurements in mm, wall thickness 3 mm.

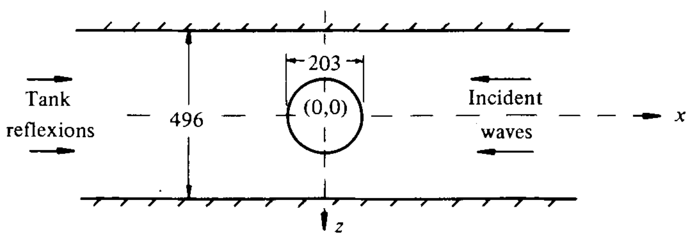

FIGURE 2. Plan view of tank.

In practice, however, the proposed scheme differs from this in being three-dimensional, and is envisaged as being composed of separate, axially-symmetric circular ducts.

The series of experiments described in this communication concern the case of a circular duct submerged beneath regular waves operating in two particular environments.

In one case experiments were conducted in a wide tank simulating the open sea, for which a theoretical treatment has recently been produced (Lighthill & Simon 1979).

In the second case experiments were conducted in a narrow tank in which diffraction and reflexion from the tank walls introduced additional effects. This corresponds in practice to the case of a single duct operating in a linear array of similar, equally-spaced ducts, seen as images in the tank walls.

2. Description of the experiments

2.1. *Narrow-tank experiments*

A duct consisting of a thin-walled hollow circular cylinder sealed at one end was immersed mouth upwards in the centre of a wave-tank; see figures 1, 2 and 3.

Waves of constant period were propagated along the channel and, after a sufficient time had elapsed for settling, wave-height recordings were made at four stations (two pairs either side of the duct). From these, the amplitudes and phases of the direct and reflected wave trains on both sides of the duct could be deduced; simultaneously a measurement was also made of the oscillating pressure in the duct. From these pressure and wave-height measurements three quantities were derived: the pressure-amplification ratio K, duct reflexion coefficient R, and pressure phase lag α.

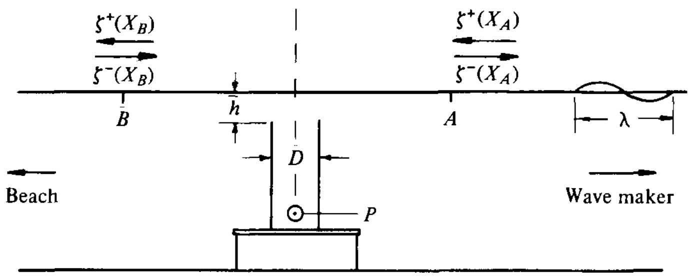

FIGURE 3. Side view of tank showing co-ordinate system and superimposed direct and reflected waves.

The character of these parameters is summarized below. However, for a full description of the method of derivation to include compensation for unwanted tank reflexions see Knott & Flower 1979.

The pressure amplification ratio K is defined as

$$K = |p|/\rho g \exp(-kh)\,|\zeta^+(X_A)^* + \zeta^-(X_B)^*|,$$

which relates the magnitude of the oscillating pressure p experienced in the depths of the duct to the magnitude which would be experienced at the level of the duct mouth owing to incident waves if the duct were removed. Here the asterisk denotes a removal of origin to the centre of the duct before vector summation of the two wave trains arriving from either side.

The reflexion coefficient R is defined as the square of the ratio of the amplitudes of the reflected and incident waves

$$R = |\zeta^-(X_A) - \zeta^-(X_B)^{**}|^2/|\zeta^+(X_A)|^2.$$

Here the double asterisk denotes that the tank reflexion $\zeta^-(X_B)$ has been attenuated and phase-shifted by transmission back across the duct before vector summation with the measured upstream reflexion.

The measured transmission coefficient which was used in this case to translate the reflected wave across the duct included implicitly a small attenuation factor due to friction in the fluid. Apart from this, however, no correction is included either here or in the derivation of the other results to account for the wave attenuation in the tank. Errors in the estimation of wave amplitude at the duct arising from this are thought to be less than 5 %, since it was observed in a study of the total energy in the incident and reflected waves, that power attenuation varied from 2 % at the lowest frequency, 0·96 Hz, to at most 10 % at 1·75 Hz between the measurement points situated 3 m apart.

The pressure phase lag α is the phase angle by which the pressure in the depths of the duct lags behind that of the incident wave.

These parameters are all defined in a two-dimensional sense. That is to say that they assume a crest-wise uniformity of the wave field. However the experiment is only nominally two-dimensional in the vicinity of the duct since the incident waves are scattered in all directions and are subsequently reflected inwards and contained within the walls. This process is, as already mentioned, equivalent to the mutual interference of an infinitely long, evenly spaced array of ducts, imaged in the walls. In this circumstance it is to be expected that the wave field in the immediate neighbourhood of the duct will be confused and non-uniform. However, from the measurement aspect the two-dimensional assumption may be deemed valid if the wave field is found to revert to a two-dimensional form at some distance from the obstruction and this was found to be the case at quite moderate distances of 2 m or so. In only one case was there an outstanding departure from this situation and this was investigated in detail; it was found that when the incident wavelength was approximately equal to the width of the tank standing waves became established across the tank. These grew and could become larger than the incident wave itself, leading to greatly increased pressures in the duct. These waves extended three-dimensionally along the length of the tank and oscillated with seemingly mixed longitudinal and transverse modes.

Records of the wave field at resonance are included to assist in visualizing the effect. Also included are lateral traverses of the tank by the wave probe in normal operation in order to illustrate the degree of crest-wise uniformity.

The presence of an anti-symmetric resonant mode at about 1·25 Hz (when the wavelength is twice the tank width) is just detectable in these traverses. However, owing to the essential symmetry of the experiment and the efficiency of the beach, very little anti-symmetric scattering was produced to reinforce this mode, and it was never observed to become large enough to materially affect the outcome of the experiment.

2.2. *Wide-tank experiments*

A series of tests were conducted in which the circular duct was immersed in the centre of a wide tank and subjected to plane incident waves. In this circumstance the influence of the tank walls could be considered negligible.

Since the tank itself was only in the early stages of commissioning the experiment could not be conducted with the same degree of precision as in the narrow tank, and so less elaborate methods for deriving reflexions and transmission were employed.

A procedure was adopted whereby a single-wave probe was used to record the wave height at the experimental station in the absence of the duct. The duct was then replaced and the oscillating pressure measured in similar conditions. The success of this method hinged on the assumption that the experimental conditions were repeatable, implying amongst other things that the scattering from the duct did not alter, over a length of time, the overall wave field in the tank. In view of its size and the efficiency of the wave-generating equipment this assumption would seem to be substantially justified. Positioning of the duct was not, however, accurate enough to allow phase information to be derived.

3. Equipment

The testing was carried out in the narrow tank at the University of Sussex and in the wide tank at the University of Edinburgh.

The principal dimensions of the narrow tank are $10 \times 0{\cdot}5$ m and 0·9 m deep. At one end of the tank waves were generated by a servo-driven Salter-type cam controlled by the output from a digital transfer-function analyser (Solartron 1600) which also performed the signal processing of the return signals. At the other end of the tank incident waves were absorbed by a wedge-shaped beach which reduced the amplitude of reflexions to less than 5 %.

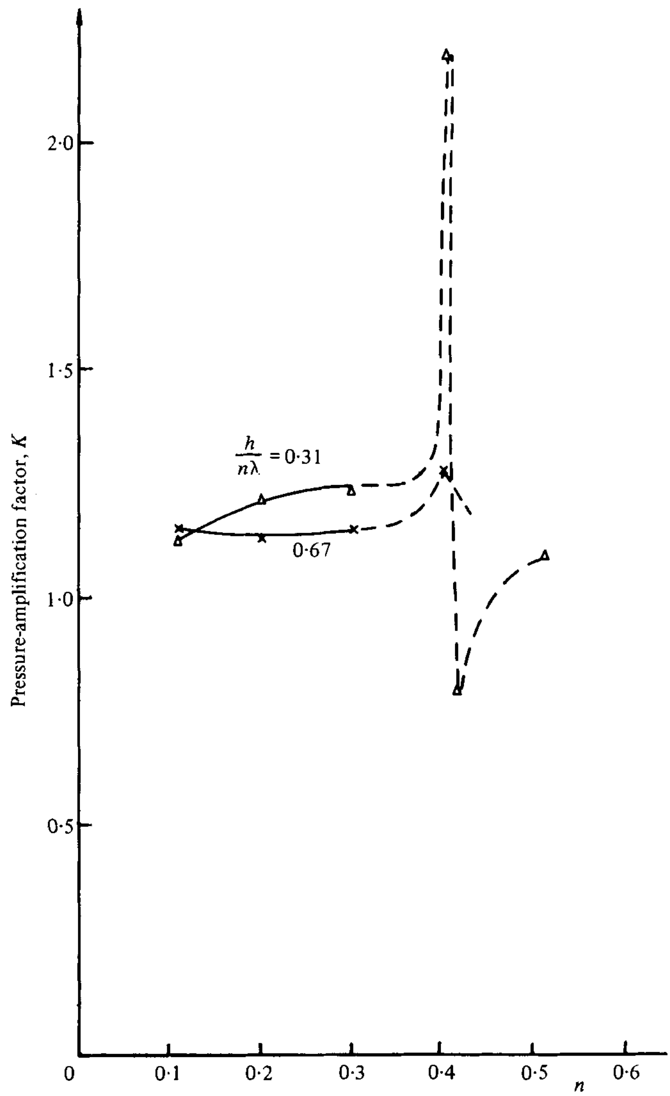

FIGURE 4. Pressure amplification factor K as a function of dimensionless duct diameter n ($n = D/\lambda$) for two values of $h/n\lambda$; narrow tank.

Wave-height measurements were made using a non-contacting gauge which allowed the wave profile to be determined across the width of the tank and across the top of the duct (Knott & Flower 1978 gives a description of this instrument). The pressure transducer was, as described in the previous work, a Gaeltec type 4T differential half-bridge strain device encapsulated in a vessel providing a constant-pressure reservoir. The reservoir was vented to the ambient pressure through a small orifice. This arrangement was a convenient way of avoiding the presence of a large hydrostatic pressure across the transducer which could have hindered the dynamic measurements. Calibration of the instrument used in this mode suggested an overall experimental accuracy of within 1 % in amplitude and 1·5° in phase.

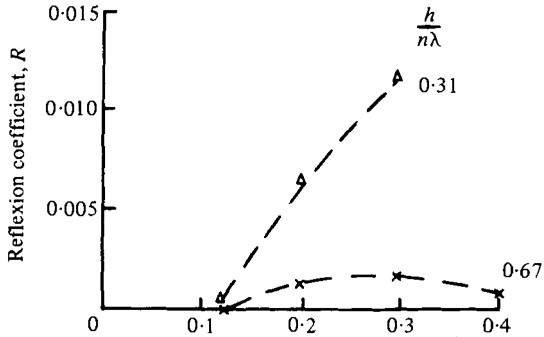

FIGURE 5. Reflexion coefficient R as a function of n for two values of $h/n\lambda$; narrow tank.

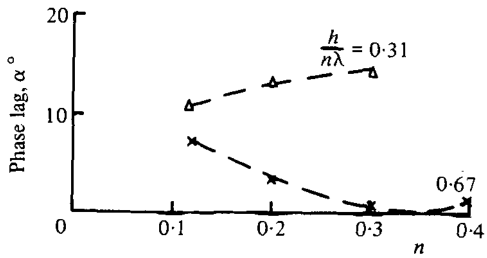

FIGURE 6. Phase lag α as a function of n for two values of $h/n\lambda$; narrow tank.

Waves in the wide tank (dimensions $25 \times 6 \times 1{\cdot}2$ m) were produced by a bank of some 90 wave-makers operated in unison to produce a uniform wave field, and these were again controlled by a Solartron transfer-function analyser. Wave measurements were made using a twin-wire probe; and the pressure transducer and duct were the same as used in the narrow-tank experiments.

4. Presentation and discussion of the results

4.1. *Narrow-tank experiments*

Figure 4 illustrates the pressure-amplification ratio K as a function of n, the ratio of the duct diameter D, to the wavelength λ. Figure 5 illustrates the reflexion coefficient, R, and figure 6 the phase lag α. Two depths of immersion, h, were employed corresponding to values of $h/n\lambda$ of 0·31 and 0·67.

The appearance of a sharp peak at the resonant frequency is quite marked in the case of the shallower depth of immersion.

Results at other frequencies can be compared with those for the two-dimensional case which are reproduced here in figure 12. (Although it should be noted the results are not entirely compatible since the latter were obtained by varying the plate spacing rather than the depth of immersion, and are expressed as a function of h/λ.)

Notwithstanding this, it is clear that the results are quantitatively similar: the pressure amplification for the three-dimensional duct is, if anything, slightly greater, the reflected power is generally lower, and the phase lags all lie between 0 and 20°.

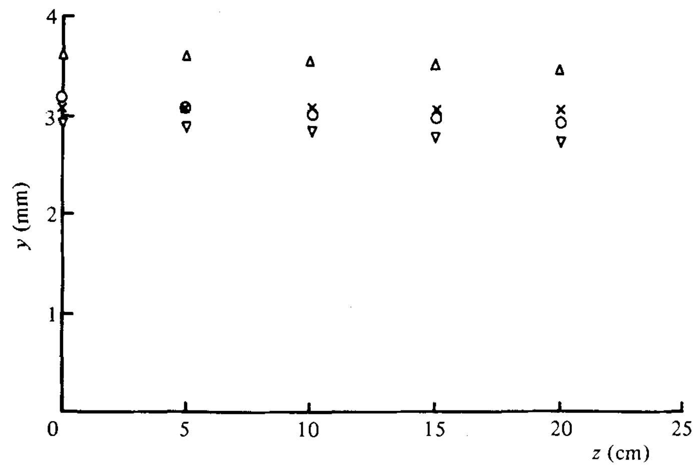

FIGURE 7. Wave-height measured across the narrow tank from the centre to the wall two metres upstream from the duct at four frequencies. $h/n\lambda = 0{\cdot}67$. Frequency (Hz): $\triangle$, 0·96; $\times$, 1·25; $\bigcirc$, 1·52; ∇, 1·75.

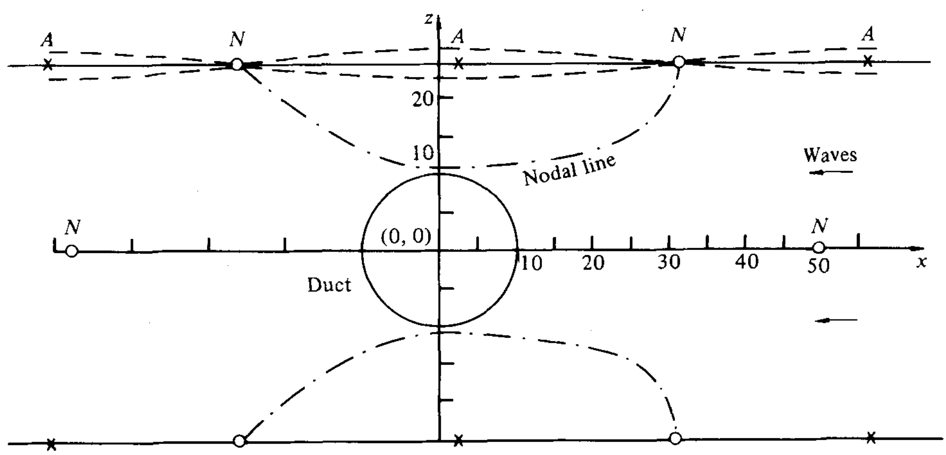

FIGURE 8. Disposition of the nodes and antinodes of the three-dimensional standing waves at resonance. $f = 1{\cdot}75$, $h/n\lambda = 0{\cdot}33$.

The result for pressure amplification is significant in predicting conditions favourable to wave-energy conversions (i.e. $K > 1$); as exist in the two-dimensional case.

A traverse of the tank revealed that the wave height as measured 2 m upstream from the duct was sensibly constant, figure 7. The highest frequency, 1·75 Hz, showed the greatest discrepancy of about 15 % between the centre of the tank and 5 cm from the side wall. The variation at 0·96 Hz is less than 6 %.

Figure 8 shows the disposition of the measured nodes and antinodes of the three-dimensional standing-wave field superimposed on the incident progressive wave at resonance: the frequency is 1·75 Hz.

The situation is illuminated further by referring to the lateral and longitudinal traverses of the wave-height measurements shown in figures 9and 10. In figures 9 (*a*, *b*)

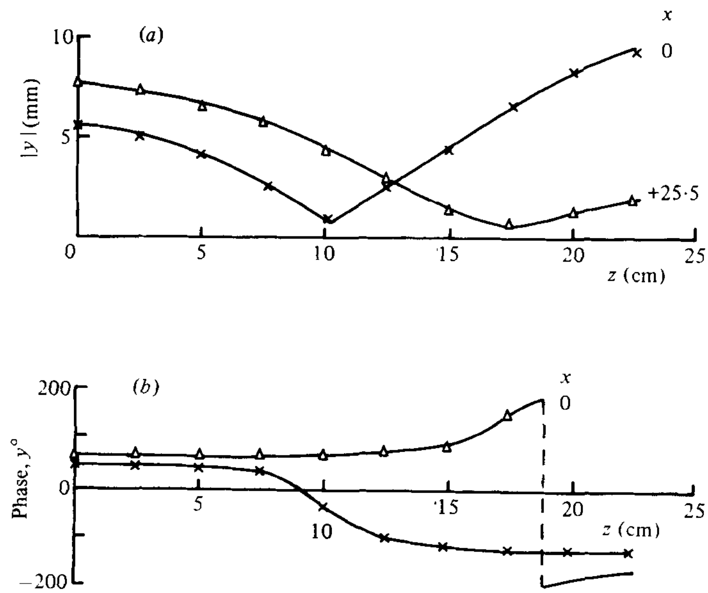

FIGURE 9. Amplitude (*a*) and phase (*b*) of the wave elevation measured across the width of the tank at resonance; $f = 1{\cdot}75$ Hz; $h/n\lambda = 0{\cdot}67$; $x = 0$ and $+25{\cdot}5$ cm.

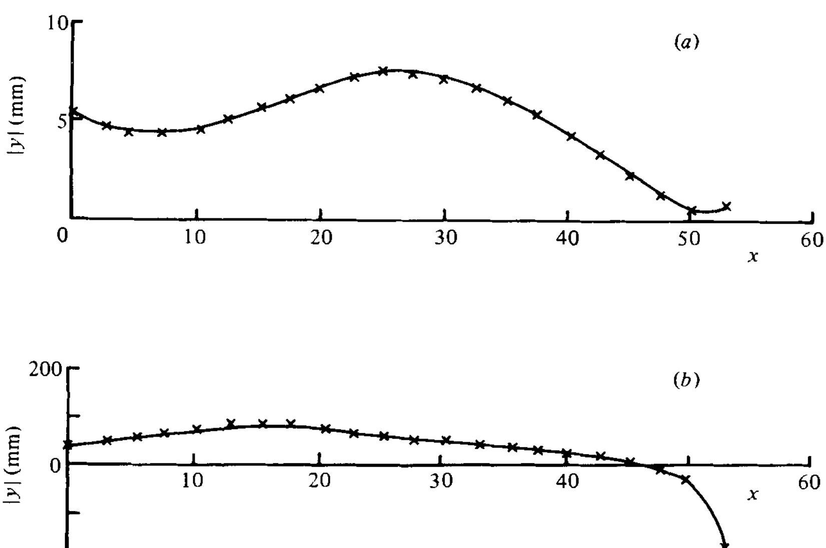

FIGURE 10. Amplitude (*a*) and phase (*b*) of the wave elevation measured upstream from the duct along the centre-line of the tank.

wave-heights and phases are shown for two lateral traverses; one through the axis of the duct (i.e. directly across it) and one through the node 25·5 cm upstream from the axis. A change of phase of 180° on passing through a node indicated the transition. This can be easily seen at the appropriate points on both of the traverses. The amplitude of the incident wave was close to 2·5 mm, hence the lateral standing wave above the cylinder can be seen to have been approximately four times this height.

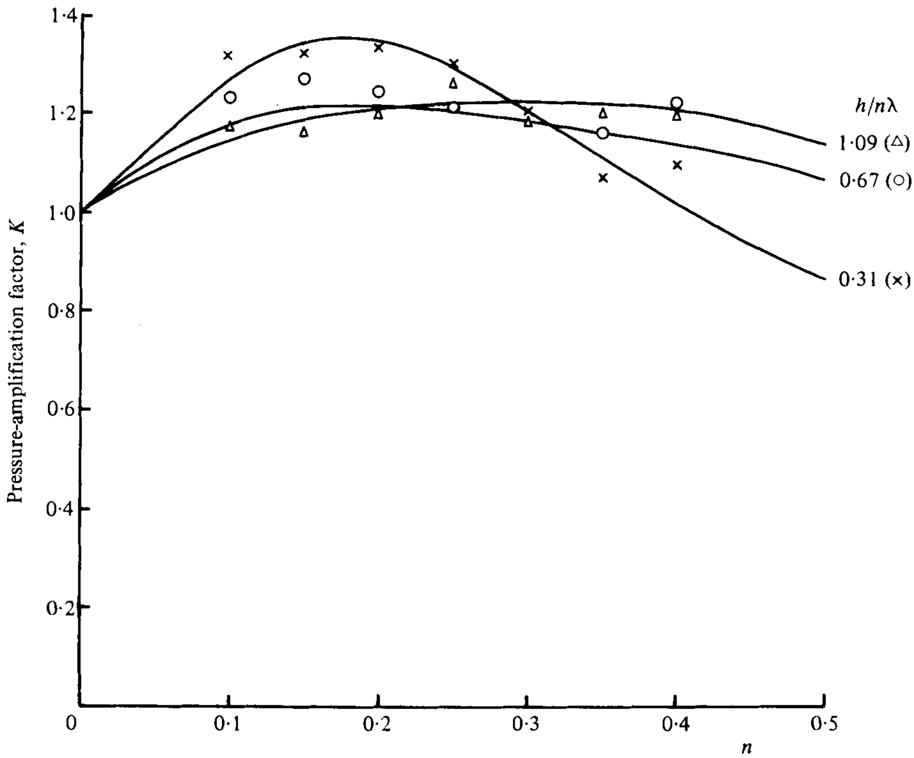

FIGURE 11. Pressure-amplification factor K as a function of n at three depths of immersion $h/n\lambda = 1{\cdot}09, 0{\cdot}67, 0{\cdot}31$ as measured in the wide tank. Also illustrated are the theoretical results of Lighthill & Simon (1979) for $h/n\lambda = 1{\cdot}09, 0{\cdot}67, 0{\cdot}31$.

In view of the relatively small scattering from the duct at other frequencies this would seem to indicate a highly resonant system. These effects were detectable in the frequency range $1{\cdot}72 \rightarrow 1{\cdot}76$ Hz only.

The longitudinal traverse upstream indicated the existence of local minima and maxima before reaching a node at approximately 50 cm upstream from the duct axis (i.e. at about one tank-width upstream).

Since this singular behaviour is uncharacteristic of the experiment in general, the close attention paid to it might appear unwarranted. However, it serves to illuminate the way in which individual three-dimensional objects may interact when placed in an array; in this case in an infinite line of equi-spaced submerged cylinders.

Interactive effects have been the subject of several recent theoretical treatments, amongst which are those of Budal (1977) and Greenhow (1979), where it has been shown that separate, individually responding absorbers in an array might, by virtue of their mutual interaction through radiation and diffraction, substantially increase their capacity to absorb energy.

The mechanism through which absorption is increased (or decreased) is intimately connected with the cross waves which build up within the array. These standing waves, which possess 'reactive' energy, do no net work but have the ability to amplify the modes of motion induced by the incident waves and thereby improve performance.

In the case considered here the cross wave is purely diffraction-generated, but it is

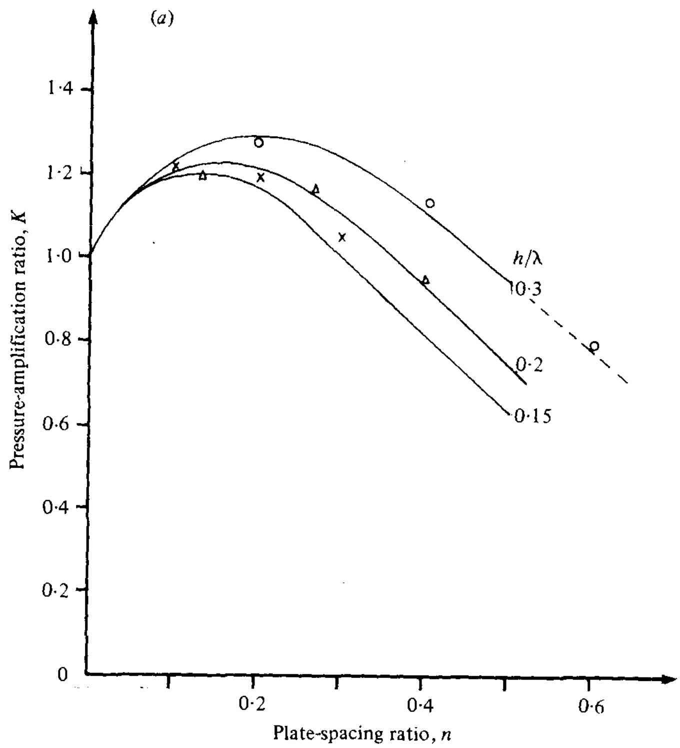

FIGURE 12(a). For legend see opposite.

to be expected that radiation-generated cross waves will assume a similar three-dimensional form.

The substantial size of this diffraction-generated wave gives a clear indication that diffraction, although less prominent in the theories for arrays than radiation, can play an important role in determining interactive effects.

4.2. *Wide-tank experiments*

The values obtained for K are plotted in figure 11 as a function of n and compared with the theoretical results due to Lighthill & Simon (1979) which were derived by an approximate variational method.

These two sets of results appear to have a fair measure of agreement although a few experiment points appeared to have strayed. These discrepancies, where they appear, do not seem to obey any consistent trend and a fair conclusion would be that the experiments substantially validate the theory to within a margin of 5 %.

5. Conclusion

Results from the wide tank indicate that pressure amplification occurs at certain wave frequencies as with the parallel-plate ducts and that the situation is well enough described by linear hydrodynamic theory. Results from the narrow tank are quali-

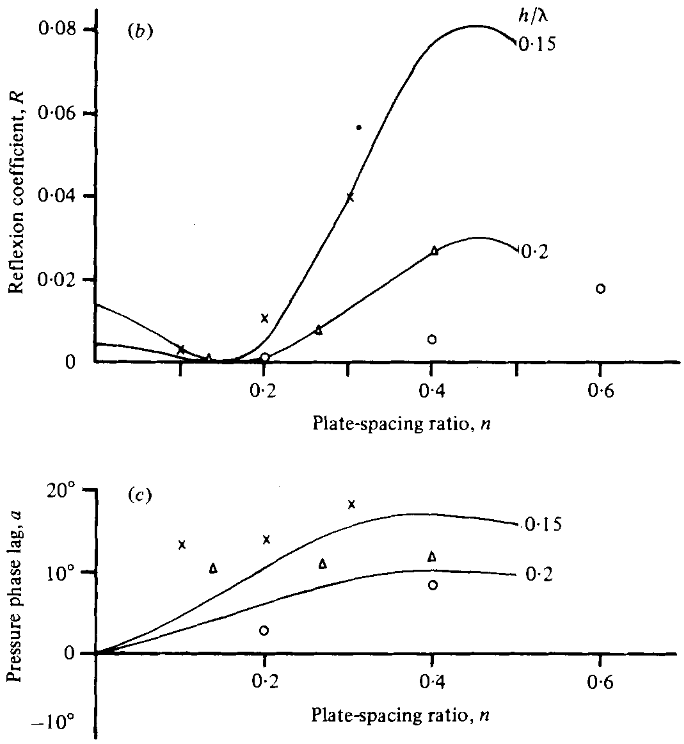

FIGURE 12. (*a*) Pressure amplification factor K, (*b*) reflexion coefficient R, and (*c*) phase lag α as functions of n for an immersed parallel-plate duct, where $n\lambda$ = distance between plates. Experiment (Knott & Flower 1979) compared with theory (Lighthill 1979).

tatively similar to the wide tank results at all but a narrow band of frequencies where resonant amplification of the cross wave occurs. The transverse wave so generated extends along the length of the tank superimposed on the progressive incident wave; the transverse wave appears extremely weakly damped and persists long after the incident driving wave is removed.

Lighthill (1979) has shown how results for the pressure amplification factor K, obtained in this way can be used to predict the performance of a device responding dynamically to waves. As such it is not of paramount importance to extend these experiments to consider the dynamic case, other than to verify the straightforward extension of theory.

However, the case for investigating dynamic performance in arrays is somewhat different. Here the situation is less amenable to theory and experiments establishing the nature of dynamic interactions will be of considerable practical importance.

The authors gratefully acknowledge the help extended by S. Salter and his colleagues at the University of Edinburgh while conducting the wide-tank experiments which were sponsored by the Wave-Energy Steering Committee, Department of Energy, and express their gratitude to M. J. Simon, University of Cambridge, for his cooperation in computing appropriate theoretical results.

Work at the University of Sussex was carried out with the financial assistance of the Marine Technology Directorate, Science Research Council.

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